Propagation of a K-body force into A-body space

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(submitted to *Phys. Rev. C*)

Abstract

The calculation of the spreading width of a compound nuclear reaction caused by a symmetry breaking K-body force acting in an A-body system ($K \ll A$, usually $K = 2$) involves the determination of the local average square matrix element in A-body space. This problem is reduced to finding the global mean square matrix element v^2 in K-body space. The result is a compact formula for the spreading width which contains v^2 as an input. Our method is based on the dilute gas approximation for excitons close to the Fermi edge. The relative strength of the contributions of operators with different exciton rank as well as the connection between the energy dependence of the spreading width and the body rank of the underlying interaction are established.

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I. INTRODUCTION

The compound nucleus (CN) has received considerable attention — both theoretical and experimental — in the recent years because of its seeming ability to enhance the effects of the weak interaction to a few percent (see for example [1] and references therein). In the present study we consider the breaking of symmetries caused by K-body forces in the compound process in general. The adequate quantity to characterize symmetry nonconservation in many-body systems is the spreading width due to the underlying interaction [2]. Although in most applications a two–body force is used, we treat the case of arbitrary K since it will allow for some insight into the energy dependence of the spreading width. This provides a generalization of the results presented in [3] and the communication of some details omitted therein.

In the statistical theory of CN reactions the matrix elements of the interaction are assumed to show the characteristics of the Gaussian orthogonal ensemble (GOE). The crucial point is that statistical assumptions of this kind can only be made about the defining matrix elements, i.e. the matrix elements in K-body space. However, the quantity of physical interest is the matrix element in A-body space. The connection between the properties of the K-body matrix elements and those in A-body space is called the propagation of the defining matrix elements [4]. In the present article a solution for this problem is offered which consists of the transition to the exciton picture (with the accompanying simplification of the basis states and complication in the description of the interaction), the propagation into the subspace of fixed exciton number, and the averaging over subspaces, which implies the return to the body picture. This procedure is necessary because our formalism makes use of the dilute gas approximation (DGA) which — in contrast to the body picture — is very good in the exciton representation of the system. In the final sections then, the general expression, which involves a convolution of partial and total densities of states, is evaluated by inserting well known analytical formulae for the level densities, the results are discussed, and limitations of our approach are indicated.

II. CONCEPTS

A. Spaces and bases

The physical quantity we are interested in is the spreading width

$$\Gamma^{\downarrow}(E) = 2\pi \ll \mathsf{V}^2 \gg \rho(E) \ . \tag{2.1}$$

Here, $\ll \mathsf{V}^2 \gg$ is the mean square matrix element of the K-body interaction V, and ρ is the level density of the system. The matrix elements are calculated in a basis of eigenstates to those parts of the Hamiltonian that dominate the behaviour of the system. The spreading width due to additional, symmetry breaking interactions then measures the extent of symmetry breaking in the system. This may include the breaking of the independent particle structure, isospin symmetry or parity by the residual strong, the electromagnetic or weak interaction, respectively. In order to properly treat the variation of $\Gamma^{\downarrow}$ with energy, the average $\ll \mathsf{V}^2 \gg$ is limited to states in the neighbourhood of some given excitation energy E. In fact, part of the present work will consist of the calculation of the strength function

$$S(E', E) \ = \ll \mathsf{V}^2 \gg \rho(E')\rho(E) \tag{2.2}$$

which implies the average over squared matrix elements between configurations close to E and configurations close to E'. Once the basis has been specified in detail, this *local* average will be defined in section III . The basis we work with is built up from the single particle states $|\lambda\rangle$ that satisfy the canonical Hartree–Fock equations

$$(\mathsf{u} + \mathsf{u}_{\rm HF})|\lambda\rangle =: \mathsf{h}_0|\lambda\rangle = \varepsilon_\lambda|\lambda\rangle \ , \lambda = 1 \ldots D \, . \tag{2.3}$$

Here, D is the dimension of the one–body space spanned by the discrete set of bound states, u is the one-body kinetic energy operator, and $\mathsf{u}_{\rm HF}$ is the Hartree-Fock mean field operator constructed from the strong nucleon-nucleon interaction, which does not include the symmetry breaking interaction V for which the spreading width shall be calculated. Let us introduce the creation operators $\mathsf{b}^\dagger_\lambda$ and the physical vacuum $|\ \rangle$ so that

$$|\lambda\rangle = \mathsf{b}^\dagger_\lambda |\ \rangle\ . \tag{2.4}$$

These operators fulfill the fundamental anticommutation relation

$$\{\mathsf{b}_\lambda\ ,\ \mathsf{b}^\dagger_{\lambda'}\} = \delta_{\lambda\lambda'}\ . \tag{2.5}$$

A basis of A-body states is then given by all possible A-fold applications of creation operators:

$$|\Lambda\rangle \equiv |\lambda_1 \ldots \lambda_A\rangle \equiv \mathsf{b}^\dagger_{\lambda_A} \ldots \mathsf{b}^\dagger_{\lambda_1} |\ \rangle\ , \tag{2.6}$$

with the condition

$$\lambda_1 < \lambda_2 < \ldots < \lambda_A. \tag{2.7}$$

The energy $\mathcal{E}_\Lambda$ of this many-body state is

$$\mathcal{E}_\Lambda = \sum_{i=1}^{A} \varepsilon_{\lambda_i}\ . \tag{2.8}$$

The energy of the ground state $|1 \ldots A\rangle$ is $\sum_{i=1}^{A} \varepsilon_i$ and will be denoted $\mathcal{E}_A$. The states (2.6) span the whole A-body space of dimension $\binom{D}{A}$, but for low energy considerations it is convenient to split the A-body space into subspaces of increasing complexity and energy and to restrict the description of the system to the subspaces in the energy range of interest. Technically this idea is realized by the transition to the exciton picture. For moderate excitation energies the state of the system does not strongly differ from the ground state and there will be only a few states occupied above and unoccupied below the Fermi energy ε_A. This will make it possible — in section III A — to introduce the so-called dilute gas approximation and treat the present problem in closed form. Let us therefore introduce the exciton creation operators [5–7] $\mathsf{a}^\dagger_\lambda$ according to [8]

$$\begin{aligned} \mathsf{a}^\dagger_\lambda &\equiv \mathsf{b}^\dagger_\lambda \qquad \text{for } \lambda > C \\ \mathsf{a}^\dagger_\lambda &\equiv \mathsf{b}_\lambda \qquad \text{for } \lambda \le C \end{aligned} \tag{2.9}$$

acting on the exciton vacuum

$$|0\rangle \equiv |1 \dots C\rangle \ . \tag{2.10}$$

One then has $\mathsf{a}_\lambda|0\rangle = 0$ for all λ and the anticommutation relation (2.5) holds for the operators a, too. Briefly: the excitons are fermions. The single body space is split into two subspaces, the single particle space with dimension $d_p \equiv D - C$ and the single hole space with dimension $d_h \equiv C$. The exciton vacuum $|0\rangle$ — the core — need not be identical to the ground state of the A-body system under consideration, i.e. at present we do not fix

$$\Delta \equiv A - C \ . \tag{2.11}$$

The energy of the vacuum is called

$$\mathcal{E}_C \equiv \sum_{\lambda=1}^{C} \varepsilon_\lambda \ . \tag{2.12}$$

Every configuration $|\Lambda\rangle$ can then be characterized by a vector

$$P \equiv (r_1, \dots, r_p) \qquad , \qquad r_i > C \tag{2.13}$$

of its particle configurations and by a vector

$$H \equiv (\alpha_1, \dots, \alpha_h) \qquad , \qquad \alpha_i \leq C \tag{2.14}$$

of its hole configurations. Every independent particle configuration $|\Lambda\rangle$ can be expressed in the form

$$|\,{}^{P}_{H}\rangle \equiv \mathsf{a}^\dagger_{r_p} \dots \mathsf{a}^\dagger_{r_1} \mathsf{a}^\dagger_{\alpha_1} \dots \mathsf{a}^\dagger_{\alpha_h} |0\rangle \ . \tag{2.15}$$

The combination of operators appearing in this equation will also be written as

$$\mathsf{a}^\dagger_{r_p} \dots \mathsf{a}^\dagger_{r_1} \mathsf{a}^\dagger_{\alpha_1} \dots \mathsf{a}^\dagger_{\alpha_h} \equiv (\mathsf{A}^P_H)^\dagger \ . \tag{2.16}$$

As usual we introduce the energies

$$\epsilon_{r_i} \equiv \varepsilon_{r_i} - \varepsilon_C \tag{2.17}$$

of the particle configurations relative to the core level as well as the energies of the hole configurations

$$\epsilon_{\alpha_i} \equiv \varepsilon_C - \varepsilon_{\alpha_i} \ . \tag{2.18}$$

The energy of the state (2.15) then is

$$\mathcal{E}_H^P \equiv \sum_{i=1}^{h} \epsilon_{\alpha_i} + \sum_{i=1}^{p} \epsilon_{r_i} + \mathcal{E}_C + \varepsilon_C \Delta \equiv \epsilon_H^P + \mathcal{E}_C + \varepsilon_C \Delta \ . \tag{2.19}$$

The symbol ϵ_H^P has been introduced to simplify the notation. In the present paper the letters r, s, t, u or v will be used for particle states and $\alpha, \beta, \gamma, \delta$ or ϵ for hole states; configurations of the physical constituents of the system — called *bodies* — are labelled λ, μ, ν, ρ or σ. If one chooses $C < A$, the ground state of the system has Δ particles and no holes, otherwise it has no particles and $-\Delta$ holes. Generally one has

$$p - h = \Delta \tag{2.20}$$

and

$$p \geq p_{\min} = \max(\Delta, 0) \ , \qquad h \geq \max(-\Delta, 0) \ . \tag{2.21}$$

The maximum number of particles is A and that of holes $C = A - \Delta$. Hence, the exciton picture provides a decomposition of the A-body space into mutually orthogonal subspaces $\mathcal{U}_p$, $p = p_{\min}, \ldots, A$. The dimension of the subspace $\mathcal{U}_p$ is $d(\mathcal{U}_p) = \binom{d_h}{h}\binom{d_p}{p}$. We use the following notation for many-exciton states: A state $|\,{}_{H'}^{P'}\rangle$ contains the same numbers of particles and holes as $|\,{}_{H}^{P}\rangle$ on possibly different single exciton states. A ket $|\,{}_{L}^{T}\rangle$ on the other hand differs from $|\,{}_{H}^{P}\rangle$ with respect to exciton number as well as single exciton states.

B. Conventions of summation

The spectral average over the squared matrix elements of V implies sums over the basis states (2.15). To avoid double counting of the basis states a definite order of the indices r and α in eq. (2.15) must be observed. We therefore introduce the notation

$$\sum_{\{H\}} \equiv \sum_{\alpha_1 < \ldots < \alpha_h} \ . \tag{2.22}$$

Here, the α_i with $i = 1, \ldots, h$ run over the configurations $1 \leq \alpha_i \leq C$. The sum is called restricted because of the restriction $\alpha_1 < \ldots < \alpha_h$ imposed on the permitted terms. Hence, the sum has $\binom{d_h}{h}$ terms. In contrast the sum

$$\sum_H \equiv \sum_{\alpha_1 \ldots \alpha_h} \tag{2.23}$$

is called unrestricted and has $(d_h)^h$ terms. Corresponding conventions are used for the set P of particle configurations. For later purposes we compress the notation of (2.22) further:

$$\sum_{\{HP;\ldots;LT\}} \equiv \sum_{\{HP\}} \ldots \sum_{\{LT\}} , \tag{2.24}$$

which means that the ordering has to be observed only within each group of indices separated by semicolons. In the following use will be made of the identity

$$\sum_{\{H\}} \langle {}^{T}_{L} | \mathsf{V} | {}^{P}_{H} \rangle^2 = \frac{1}{h!} \sum_{H} \langle {}^{T}_{L} | \mathsf{V} | {}^{P}_{H} \rangle^2 , \tag{2.25}$$

which holds because the *squared* matrix element is completely symmetric with respect to the h indices in H, but vanishes if any two of them coincide.

C. Interactions for bodies and excitons

Now that the basis we are going to work with is specified, we turn to those parts of the interaction that have not been considered in its determination and consequently cause transitions between basis states. Apart from the residual strong interaction this includes the electromagnetic, weak, and other possible forces. In occupation number formalism, a K-body operator

$$\mathsf{V} = \sum_{i_1 < \ldots < i_K = 1}^{A} \mathsf{v}(x_{i_1} \ldots x_{i_K}) \tag{2.26}$$

has the form

$$\mathsf{V} = \sum_{\{\mu_1,\ldots,\mu_K;\, \nu_1,\ldots,\nu_K\}} \langle \mu_1 \ldots \mu_K | \tilde{\mathsf{v}} | \nu_1 \ldots \nu_K \rangle \mathsf{b}^\dagger_{\mu_K} \ldots \mathsf{b}^\dagger_{\mu_1} \mathsf{b}_{\nu_1} \ldots \mathsf{b}_{\nu_K} , \tag{2.27}$$

with the totally antisymmetric matrix element $\langle\mu_1 \ldots \mu_K|\tilde{\mathsf{v}}|\nu_1 \ldots \nu_K\rangle$. How is this representation affected by the transition to the exciton picture? The range of summation of every index is split into two parts, body operators are replaced by exciton operators according to eq. (2.9), and the resulting terms are brought into normal order and grouped according to particle-hole structure. For $K = 2$, the result is given in refs. [5,7,9] and is reproduced in Tab. I. The interaction is the sum of the fourteen terms ${}_k\mathsf{V}_{(a,q)}$ listed there together with their Feynman diagrams [9]. The diagrams facilitate the visualization of the systematics in and the generalization of the contents of Tab. I, see Fig. 1 . The exciton operators can be classified by the three numbers k, a, and q. The rank k of ${}_k\mathsf{V}_{(a,q)}$ is half the number of external lines in the corresponding diagram. One finds $0 \leq k \leq K$. The parameter a is the number of particle-hole pairs created by ${}_k\mathsf{V}_{(a,q)}$: the interaction does not conserve the number of excitons; the conservation of the number of physical bodies, however, requires that the exciton number changes by particle-hole pairs. The range of a obviously is $-k \leq a \leq k$. Finally, q is the number of upward arrows in the diagram. One recognizes that q changes in steps of two, since for fixed a every additional particle before the interaction leads to an additional particle after the interaction. The range of q is found to be $|a| \leq q \leq 2k - |a|$. Hence,

$$\mathsf{V} = \sum_{k=0}^{K} \sum_{a=-k}^{k} \sum_{\substack{q=|a| \\ \Delta q=2}}^{2k-|a|} {}_k\mathsf{V}_{(a,q)}. \tag{2.28}$$

On the table, the operators with $k < 2$ have contracted hole lines that represent the interaction of the excitons with the nuclear core. It is easily seen (and holds for arbitrary K as well) that the number of particle and hole lines before and after the interaction is given by $N_- \equiv k + a$ and $N_+ \equiv k - a$, respectively. Out of these there are $p_\pm \equiv \frac{q \pm a}{2}$ particles and $h_\pm \equiv N_\pm - p_\pm = k - p_\mp$ holes. We give a few identities that clarify the significance of these quantities:

$$N_+ + N_- = 2k$$

$$p_+ + p_- = q$$

$$
\begin{aligned}
& h_+ + h_- = 2k - q \\
& p_+ - p_- = h_+ - h_- = \frac{1}{2}(N_+ - N_-) = a \\
& p_+ - h_+ = p_- - h_- = q - k \\
& p_- + h_+ = p_+ + h_- = k \ .
\end{aligned}
\tag{2.29}
$$

Furthermore, we introduce $h_C \equiv K - k$, which is the number of contracted hole indices of ${}_k\mathsf{V}_{(a,q)}$. The contracted indices never appear with exciton operators. Since as a consequence of (2.9) hole indices associated with creators (annihilators) appear in the bra (ket) of the matrix element, the general structure of ${}_k\mathsf{V}_{(a,q)}$ is [10]:

$$
\begin{aligned}
{}_k\mathsf{V}_{(a,q)} = & \sum_{\{\alpha_1 \ldots \alpha_{h_C}\}} \sum_{\{\beta_1 \ldots \beta_{h_-}\}} \sum_{\{s_1 \ldots s_{p_-}\}} \sum_{\{\gamma_1 \ldots \gamma_{h_+}\}} \sum_{\{t_1 \ldots t_{p_+}\}} \\
& \langle \alpha_1 \ldots \alpha_{h_C} \beta_1 \ldots \beta_{h_-} t_1 \ldots t_{p_+} | \tilde{\mathsf{v}} | \alpha_1 \ldots \alpha_{h_C} \gamma_1 \ldots \gamma_{h_+} s_1 \ldots s_{p_-} \rangle \\
& \times \mathsf{a}^\dagger_{t_{p_+}} \ldots \mathsf{a}^\dagger_{t_1} \mathsf{a}^\dagger_{\gamma_1} \ldots \mathsf{a}^\dagger_{\gamma_{h_+}} \mathsf{a}_{\beta_{h_-}} \ldots \mathsf{a}_{\beta_1} \mathsf{a}_{s_1} \ldots \mathsf{a}_{s_{p_-}} \\
= & \sum_{\{H_C;\, H_- P_-;\, H_+ P_+\}} \langle H_C H_- P_+ | \tilde{\mathsf{v}} | H_C H_+ P_- \rangle \left(\mathsf{A}^{P_+}_{H_+} \right)^\dagger \left(\mathsf{A}^{P_-}_{H_-} \right) \ .
\end{aligned}
\tag{2.30}
$$

D. Partial and total level densities

In the statistical model of CN processes, reaction rates are dominated by the available phase space. This calls for a detailed knowledge of the densities of states, which have been the subject of intensive studies, see for example [11–14]. In the present section, we quote the results relevant for the sequel. The density of states for the A-body system,

$$
\rho(\mathcal{E}) = \sum_{\{\Sigma\}} \delta(\mathcal{E} - \mathcal{E}_\Sigma) \ , \tag{2.31}
$$

is approximated by continuous expressions derived with methods of statistical mechanics. The famous Bethe formula is

$$
\rho^{(\mathrm{B})}(E) = \frac{\exp[2(\pi^2 g E/6)^{1/2}]}{\sqrt{48} E}. \tag{2.32}
$$

Here, E is the *excitation* energy of the system, $E \equiv \mathcal{E} - \mathcal{E}_A$ (with the ground state energy $\mathcal{E}_A$) and g is the single body density at the Fermi edge. Blatt and Weisskopf [15] give the expression

$$\rho^{(\mathrm{BW})}(E) = C\exp[2(aE)^{1/2}] \ , \tag{2.33}$$

which may be understood as an approximation to eq. (2.32) in so far as it contains only the term varying most rapidly with energy. Finally the approximation of constant temperature (CTA) yields a purely exponential increase for the nuclear level density, i.e.

$$\rho^{(\mathrm{CTA})}(y) = \rho(y_0)\mathrm{e}^{(y-y_0)/T} \ . \tag{2.34}$$

Here we have called the excitation energy y, because we will need the total level density in this form in section IV B. The nuclear temperature is defined as

$$T = \sqrt{\frac{6y_0}{\pi^2 g}} \ . \tag{2.35}$$

Gilbert and Cameron [12] and v. Egidy and collaborators [16] have compared these expressions with experimental data on nuclear level densities and found that the CTA gives a good fit up to excitation energies of approximately 10 MeV, whereas above this value the Bethe or Blatt and Weisskopf expressions must be used.

Since we are going to work with the exciton picture, we need expressions for the densities of states with fixed exciton number. They will be characterized by the number of holes and particles that occur:

$$\sum_{\{HP\}} \delta(E - E_H^P) \equiv \rho_{h,p}(E) = \rho_{p-\Delta,p}(E) \ . \tag{2.36}$$

Here, $E_H^P = \mathcal{E}_H^P - \mathcal{E}_A = \epsilon_H^P + \mathcal{E}_C + \varepsilon_C\Delta - \mathcal{E}_A \equiv \epsilon_H^P + \mathcal{S}$. The shift $\mathcal{S}$ is nonzero only if $C \neq A$. Summation over the subspaces yields the total density of states:

$$\sum_{p=\max(\Delta,0)}^{A} \rho_{p-\Delta,p}(E) = \rho(E) \ . \tag{2.37}$$

The identity (2.37) is of course independent of Δ. A different Δ merely implies a different description, but does not alter the physics of the level density of the A-body system:

$$\sum_{p=\max(\Delta,0)}^{A} \sum_{\{P-\Delta,P\}} \delta(E - E^P_{P-\Delta}) = \sum_{p=\max(\Delta',0)}^{A} \sum_{\{P-\Delta',P\}} \delta(E - E^P_{P-\Delta'}) = \rho(E) \ . \qquad (2.38)$$

Since we are going to make use of this invariance property of ρ in section III B, it is necessary to explain in detail how it is to be understood. The sum over p on the r.h.s. of eq. (2.38) may contain subspaces (e.g. $1p\ 3h$ configurations) that do not appear at all on the left hand side (which could start with the subspace of $2p\ 0h$ configurations). And also the single exciton spaces are different: an increase of Δ decreases the dimension of the single hole space and enlarges that of the single particle space. If we assume a constant spacing of the single body levels, however, the single exciton energies ϵ_{r_i} and ϵ_{α_i} in eqs. (2.17) and (2.18) that occur in the summations over the subspace configurations are not affected by the transformation (2.38) — except for the highest excited states. We can safely ignore this difference in the energy range of interest. Explicitly, the identity (2.38) then reads

$$\sum_{p=\max(\Delta,0)}^{A} \sum_{\{P-\Delta,P\}} \delta(E - \epsilon^P_{P-\Delta} - \mathcal{S}) = \sum_{p=\max(\Delta',0)}^{A} \sum_{\{P-\Delta',P\}} \delta(E - \epsilon^P_{P-\Delta'} - \mathcal{S}') \ , \qquad (2.39)$$

where $\mathcal{S}' = \mathcal{E}_{C'} + \varepsilon_{C'}\Delta' - \mathcal{E}_A$, see below eq. (2.36). Ericson [11] gave the well-known approximate expression

$$\rho^{(\mathrm{Er})}_{h,p}(E) = \frac{g[g(E-\mathcal{S})]^{h+p-1}}{h!p!(h+p-1)!} \ , \qquad (2.40)$$

for the partial density of states (2.36). It is valid if the Pauli principle for excitons is ignored. This corresponds to the "dilute gas approximation" discussed below. Eq. (2.40) was lateron improved in many respects (e.g. [17,18]), but is widely used because of its simplicity.

III. PROCEDURE

A. Propagation into the Subspace

The strength function for transitions between states of energy E and exciton number $N = p + h$ and states of energy E' and exciton number $N' = t + l$ is defined as

$$S_{(t,p)}(E',E) = \sum_{\{LT;\,HP\}} \langle {}^{T}_{L}|\mathsf{V}|{}^{P}_{H}\rangle^2 \delta(E'-E^T_L)\delta(E-E^P_H). \tag{3.1}$$

The local average $<\mathsf{V}^2>$ over the squared matrix elements between states with E, N and E', N' then is

$$<\mathsf{V}^2>\rho_{l,t}(E')\rho_{h,p}(E) = S_{(t,p)}(E',E)\ , \tag{3.2}$$

where the partial densities of states are defined in eq. (2.36). Hence, the average is defined with the delta functions of the energies as weighting factors. However, in the present context $\delta(E-E^P_H)$ is to be understood not as the Dirac distribution but rather as a peaked function of suitable width. The "suitable width" is large compared to the average level spacing and small compared to intervals over which secular variations of the level densities occur. This guarantees that densities of states and strength functions are smooth and can be reasonably approximated by the expressions of subsection II D. The strength produced by the interaction operator ${}_k\mathsf{V}_{(a,q)}$ is by specialization of eq. (3.1) and use of eq. (2.30):

$$\begin{aligned} {}^kS^{(a,q)}_{(t,p)}(E',E) &\equiv \sum_{\{LT;\,HP\}} \langle {}^{T}_{L}|\ {}_k\mathsf{V}_{(a,q)}|{}^{P}_{H}\rangle^2 \delta(E'-E^T_L)\delta(E-E^P_H) \qquad (3.3)\\ &= \sum_{\{LT;\,HP\}} \langle {}^{T}_{L}| \Bigg[\sum_{\{H_C;\,H_+P_+;H_-P_-\}} \langle H_C H_- P_+|\tilde{\mathsf{v}}|H_C H_+ P_-\rangle \left(\mathsf{A}^{P_+}_{H_+}\right)^\dagger \left(\mathsf{A}^{P_-}_{H_-}\right) \Bigg] |{}^{P}_{H}\rangle^2 \delta(E'-E^T_L)\delta(E-E^P_H). \end{aligned}$$

Note that of the seven quantities that specify the strength only six are independent since $t = p+a$. Furthermore we observe that a nonzero contribution comes only from the subspaces with $p \geq p_-$ and $h \geq h_-$, because otherwise the matrix element vanishes. We proceed to estimate this expression in three steps.

(i) In the first step the K-body matrix elements of V are considered to be entries of a random matrix. Invoking time reversal invariance we postulate that V (in K-body space) belongs to the Gaussian Orthogonal Ensemble (GOE). This means that the second moments of the matrix elements in eq. (2.27) can be expressed by a single parameter v^2, namely (a bar over a symbol denotes the ensemble average)

$$\overline{\langle K_1|\tilde{\mathsf{v}}|K_2\rangle\langle K_1'|\tilde{\mathsf{v}}|K_2'\rangle} = v^2[\delta^{K_1'}_{K_1}\delta^{K_2'}_{K_2} + \delta^{K_2'}_{K_1}\delta^{K_1'}_{K_2}] \tag{3.4}$$

for K-body configurations K_1, K_2, K_1', K_2'. Here, $\delta^K_{K'}$ is a generalized Kronecker symbol [19] with the properties

$$\delta^K_{K'} = \begin{cases} 1 & : \text{ if } K' \text{ is an even permutation of } K \\ -1 & : \text{ if } K' \text{ is an odd permutation of } K \\ 0 & : \text{ if two indices out of } K \text{ or out of } K' \text{ coincide} \\ 0 & : \text{ if } K' \text{ is not a permutation of } K \ . \end{cases} \tag{3.5}$$

This allows us to estimate the strength function (3.3) by its ensemble average, or — by the same token — the spectral average $< \mathsf{V}^2 >$ is identified with the ensemble average.

(ii) How does the correlation rule (3.4) translate into the exciton picture? We find the following relation:

$$\overline{\langle H_C H_- P_+ | \tilde{\mathsf{v}} | H_C H_+ P_- \rangle \langle L_C L_- T_+ | \tilde{\mathsf{v}} | L_C L_+ T_- \rangle} = v^2 [\delta^{L_C L_- T_+}_{H_C H_- P_+} \delta^{L_C L_+ T_-}_{H_C H_+ P_-} + \delta^{L_C L_+ T_-}_{H_C H_- P_+} \delta^{L_C L_- T_+}_{H_C H_+ P_-}]. \tag{3.6}$$

In principle, eq. (3.6) allows the simplification of the various terms in the strength function (3.1). The evaluation of the sum over the hole indices $\{H_C; H_+; H_-\}$ in eq. (3.3), however, is complicated by the fact that it is not completely restricted in the sense of eq. (2.22). We therefore use an approximation to eq. (3.6):

$$\overline{\langle H_C H_- P_+ | \tilde{\mathsf{v}} | H_C H_+ P_- \rangle \langle L_C L_- T_+ | \tilde{\mathsf{v}} | L_C L_+ T_- \rangle} = v^2 \delta^{L_C}_{H_C} [\delta^{L_- T_+}_{H_- P_+} \delta^{L_+ T_-}_{H_+ P_-} + \delta^{L_+ T_-}_{H_- P_+} \delta^{L_- T_+}_{H_+ P_-}]. \tag{3.7}$$

The quality of this approximation is discussed in appendix A, where the additional correlations resulting from eq. (3.6) are found to be negligible. Note that rule (3.7) implies that different exciton operators are uncorrelated. Therefore the cross terms appearing in (3.1) do not contribute to the strength function, which may consequently be obtained by summing eq. (3.3) over k, a and q. With the approximate correlation rule one easily arrives at:

$${}^k S^{(a,q)}_{(t,p)}(E', E) = v^2 \sum_{\{H_C; H_+ P_+; H_- P_-\}} \sum_{\{LT; HP\}} \langle {}^T_L | \left(\mathsf{A}^{P_+}_{H_+} \right)^\dagger \left(\mathsf{A}^{P_-}_{H_-} \right) | {}^P_H \rangle^2 \delta(E' - E^T_L) \delta(E - E^P_H) \ . \tag{3.8}$$

The contribution of the second term on the r.h.s. of eq. (3.7) is restricted to the diagonal elements of the matrix and is therefore neglected relative to the first one. By eq. (3.8)

the problem factors into two aspects: All information that is specific for the interaction is contained in v^2. In the present paper this factor is taken for granted. The remaining sum is common to all interactions of body rank K. It represents the phase space aspect of the problem, the propagation of the K-body force into the A-body space.

The summation over H_C, which appears for all operators with $k < K$ is now trivial and yields the factor $\binom{d_h}{h_C}$.

(iii) The sum over $\{LT; PH\}$ is rewritten in a form which one can call the separation of actors and spectators. The configurations $\{H_+P_+\}$ and $\{H_-P_-\}$ that appear in the operators A are called actors. They must appear in $\langle \begin{smallmatrix} T \\ L \end{smallmatrix}|$ and $| \begin{smallmatrix} P \\ H \end{smallmatrix}\rangle$, respectively, if the matrix element in eq. (3.8) is to be different from zero. The remaining configurations that are possibly present in $\langle \begin{smallmatrix} T \\ L \end{smallmatrix}|$ and $| \begin{smallmatrix} P \\ H \end{smallmatrix}\rangle$ are called spectators. As proven in appendix B one finds

$$\sum_{\{LT;\,HP\}} \langle \begin{smallmatrix} T \\ L \end{smallmatrix}| \left(\mathsf{A}^{P_+}_{H_+}\right)^\dagger \left(\mathsf{A}^{P_-}_{H_-}\right) | \begin{smallmatrix} P \\ H \end{smallmatrix}\rangle^2 \quad f(L,T,H,P)$$
$$= \sum_{\substack{\{H-h_-,P-p_-\} \\ \neq H_-P_-H_+P_+}} f((H-h_-,H_+),(P-p_-,P_+),(H-h_-,H_-),(P-p_-,P_-)) \;, \tag{3.9}$$

where f is any function that is completely symmetric in the indices contained in L, T, H and P (for each group separately). The sum on the r.h.s. of eq. (3.9) runs over the spectators. The notation $\{H-h_-, P-p_-\}$ means a string of $h-h_-$ indices of holes and a string of $p-p_-$ indices of particles that observe the restrictions of eq. (2.22). In eq. (3.9), there is the additional restriction $\neq H_-P_-H_+P_+$ which means that none of the indices in $\{H-h_-, P-p_-\}$ is allowed to coincide with any one of the indices in $H_-P_-H_+P_+$. By help of eq. (3.9) one can simplify eq. (3.8) as follows

$${}^kS^{(a,q)}_{(t,p)}(E',E) = v^2\binom{d_h}{h_C} \sum_{\{H_+P_+;\,H_-P_-\}} \sum_{\substack{\{H-h_-,P-p_-\} \\ \neq H_+P_+H_-P_-}} \delta(E' - E^{P-p_-}_{H-h_-} - \epsilon^{P_+}_{H_+}) \delta(E - E^{P-p_-}_{H-h_-} - \epsilon^{P_-}_{H_-})$$

$$= v^2\binom{d_h}{h_C} \sum_{\{H_+P_+;\,H_-P_-\}} \sum_{\substack{\{H-h_-,P-p_-\} \\ \neq H_+P_+H_-P_-}} \int \mathrm{d}y \; \delta(E' - y - \epsilon^{P_+}_{H_+}) \delta(E - y - \epsilon^{P_-}_{H_-}) \delta(y - \epsilon^{P-p_-}_{H-h_-} - \mathcal{S}), \tag{3.10}$$

where we have used $E^{P-p_-}_{H-h_-} = \epsilon^{P-p_-}_{H-h_-} - \mathcal{S}$. As already mentioned, the sum over the spectators is not independent of the sum over the actors: The spectators are not allowed to occupy the

exciton states of the actors. This is only a weak condition if the dimensions of the single particle and single hole spaces are much larger than the number of excitons that occur. If the energies E and E' are not too high, this will be true and one can treat the sums in eq. (3.10) as independent. This is called the dilute gas approximation (DGA). It allows us to express the strength function by the convolution of partial densities of states

$$ {}^{k}S^{(a,q)}_{(t,p)}(E',E) = v^2 \binom{d_h}{h_C} \int \mathrm{d}y \, \rho_{h_+,p_+}(E'-y+\mathcal{S}) \rho_{h_-,p_-}(E-y+\mathcal{S}) \rho_{h-h_-,p-p_-}(y) \ . \quad (3.11) $$

As discussed in detail in appendix A, the conditions for the validity of the DGA and for the applicability of the approximate correlation rule (3.7) are essentially the same.

B. Propagation into the A-body space

In section III A, the matrix element was averaged over configurations with given exciton numbers. This result is useful if pre-equuilibrium reactions are studied. In equilibrium CN reactions one asks for the average $\ll \mathsf{V}^2 \gg$ over the full A-body space which is defined as

$$ \begin{aligned} S\,(E',E) &\equiv \ll \mathsf{V}^2 \gg \rho\ (E') \rho\ (E) \qquad (3.12) \\ &= \sum_{t,p=\max(\Delta,0)}^{A} \sum_{\{LT;\,HP\}} \langle {}^{T}_{L} | \mathsf{V} | {}^{P}_{H} \rangle^2 \delta(E'-E^T_L) \delta(E-E^P_H) \end{aligned} $$

in close analogy with eqs. (3.1) and (3.2). The summation of the partial strengths up to $p,t=A$ is formally correct although in the energy range we are interested in (and committed to because of the DGA) by far not all subspaces come into play. The high energy subspaces are excluded by the multiplication with δ-functions. As discussed in the last section and in appendix A, different exciton operators are uncorrelated in the framework of the DGA. We therefore obtain the strength $S(E',E)$ as a sum of the contributions by the operators ${}_{k}\mathsf{V}_{(a,q)}$:

$$ S(E',E) = \sum_{kaq} \quad {}^{k}S^{(a,q)}(E',E) \ , \qquad (3.13) $$

in obvious notation. These contributions, in turn, are easily expressed by the strength functions (3.3):

$$ {}^kS^{(a,q)}(E',E) = \sum_{p=\max(\Delta+k-\frac{q+a}{2},\frac{q-a}{2})}^{\min(A,A-a)} {}^kS^{(a,q)}_{(p+a,p)}(E',E)\,, \tag{3.14} $$

where the condition $t = p + a$ has been used to evaluate one of the sums over the subspaces. The lower limit of the remaining sum guarantees that $p \geq p_-$, $h \geq h_-$ (see below eq. (3.3)) and $p \geq \Delta$ (see eq. (2.20)). The upper limit ensures that $p, t \leq A$. We introduce the index of summation $i = p - \frac{q-a}{2}$ and the strength becomes:

$$ {}^kS^{(a,q)}(E',E) = \sum_{i=\max(\Delta+k-q,0)}^{A-\frac{q+|a|}{2}} {}^kS^{(a,q)}_{(i+p_+,i+p_-)}(E',E)\ . \tag{3.15} $$

The second form of eq. (3.10) then yields

$$ \begin{aligned} {}^kS^{(a,q)}(E',E) = v^2 \binom{d_h}{h_C} \sum_{\substack{i=\max \\ (\Delta+k-q,0)}}^{A-\frac{q+|a|}{2}} \sum_{\{I-\Delta-k+q,I\}} \int \mathrm{d}y\, \rho_{h_+,p_+}(E'-y+\mathcal{S})\rho_{h_-,p_-}(E-y+\mathcal{S}) \\ \times\, \delta(y-\epsilon^I_{I-\Delta-k+q}-\mathcal{S})\ . \end{aligned} \tag{3.16} $$

Here, the partial level densities of actors are the same as those in eq. (3.11). In eq. (3.16) the expression

$$ \mathrm{R}(y) \equiv \sum_{p=\max(\Delta',0)}^{A'} \sum_{\{P-\Delta',P\}} \delta(y-\epsilon^P_{P-\Delta'}-\mathcal{S})\ . \tag{3.17} $$

appears with $A' = A - \frac{q+|a|}{2}$ and $\Delta' = \Delta + k - q \equiv \Delta + z$. We want to compare R with the total density of states ρ of the A-body system, the definition (2.37) of which is quite similar to the expression (3.17). The comparison with $\rho(y)$ is possible if we exploit the invariance of the nuclear level density under shifts of the exciton vacuum. We recall eq. (2.39):

$$ \rho(y) = \sum_{p=\max(\Delta',0)}^{A} \sum_{\{P-\Delta',P\}} \delta(y-\epsilon^P_{P-\Delta'}-\mathcal{S}')\ . \tag{3.18} $$

Expressions (3.18) and (3.17) differ in two respects :

1. The subspaces with $p = A' + 1 \ldots A$ do not appear in eq. (3.17).

2. The argument is shifted by $\mathcal{S}' - \mathcal{S}$.

In the energy range of typical CN reactions the first point is irrelevant so that the only remaining difference is the shift of the argument:

$$\mathrm{R}(y) = \rho(y + \mathcal{E}_{C'} - \mathcal{E}_C + \varepsilon_{C'}\Delta' - \varepsilon_C\Delta) \tag{3.19}$$

In the approximation of equidistant single body levels we find the relation:

$$R(y) = \rho\left(y - \frac{z}{g}\left(\frac{z+1}{2} + \Delta\right)\right). \tag{3.20}$$

At first sight one may be surprised to find that this relation depends on Δ, which *a priori* we may choose arbitrarily. On the other hand, however, the choice of Δ determines the quality of the DGA: it is best if those subspaces that contribute most to the strength are made up of as few excitons as possible. The more states are excluded by the Pauli–principle, the larger is the error in eq. (3.16). Evaluating the integral, we choose $\Delta = 0$, which optimizes the DGA. Hence,

$${}^kS^{(a,q)}(E', E) = v^2 \binom{d_h}{h_C} \int \mathrm{d}y\, \rho_{h_+,p_+}(E' - y)\rho_{h_-,p_-}(E - y)\rho\left(y - E_z\right) \ , \tag{3.21}$$

with

$$E_z = \frac{z(z+1)}{2g} \ , \tag{3.22}$$

which is the energy needed to create z particles (negative z corresponds to the creation of holes).

IV. RESULTS

A. Transition rates in the exciton model

Formula (3.11) can be applied to the exciton model of preequilibrium nuclear reactions. In this model, which was formulated by Griffin [20] and has later been refined by several authors [21–24], an important concept is that of transition rates between subspaces of different exciton number. These appear in the Master equation for the time dependence of

the system. Transitions are assumed to be caused by the residual interaction of a strong two-body force V. The rate for going from the subspace $\mathcal{U}_p$ to $\mathcal{U}_{p+a}$ is given by

$$\lambda_{p\to p+a}(E) = \frac{2\pi}{\hbar} < \mathsf{V}^2 > \rho_{h+a,p+a}(E) \tag{4.1}$$

and is related to the strength calculated in section III A according to

$$\lambda_{p\to p+a}(E)\rho_{h,p}(E) = \frac{2\pi}{\hbar}\sum_q \quad {}^2S^{(a,q)}_{(p+a,p)}(E,E) \ . \tag{4.2}$$

Note that the residual interaction consists by definition of the operators with exciton rank $k = 2$ that appear if V is expressed in the exciton picture using the Hartree-Fock single exciton configurations. Invoking the Ericson densities (2.40) and $\Delta = 0$, the convolution (3.11) can easily be evaluated and leads to the transition rate

$$\lambda_{p\to p+a}(E) \ = \ \frac{2\pi}{\hbar}v^2 \sum_q \ {}^2\rho^{(a,q)}_{h,p}(E) \ , \tag{4.3}$$

with the density of final states

$${}^k\rho^{(a,q)}_{h,p}(E) = \ \binom{d_h}{h_C}\binom{p}{p_-}\binom{h}{h_-}\binom{N+k+a-2}{N-1}^{-1}\binom{2k-2}{k+a-1}\ \rho_{h_+,p_+}(E) \ , \tag{4.4}$$

where $N = p + h$, see above eq. (3.1). Because of the last binomial only operators with $k > |a|$ contribute. The final state densities that appear here in the context of propagation of the defining GOE matrix elements have been obtained earlier for $k = 2$ by combinatorial arguments on the states accessible in two-body collisions [25,26,18]. Originally, v^2 was a fit parameter. It is identified here as the average square of the antisymmetric K-body matrix element.

B. Spreading width in A-body space

Formula (3.21) can be evaluated by inserting Ericson's expressions for the partial and the CTA for the total density of states, see eqs. (2.40) and (2.34) . The operator ${}_k\mathsf{V}_{(a,q)}$ then yields the strength:

$$ {}^{k}S^{(a,q)}(E) = v^2 \binom{d_h}{h_C} \binom{2k-2}{k+a-1} \frac{g(gT)^{2k-1}}{p_+!h_+!p_-!h_-!} \quad \exp\left(-\frac{z(z+1)}{2gT}\right) \ \rho(E). \tag{4.5} $$

We have chosen $y_0 = E$ in eq. (2.34). As mentioned in section II D, below $E \approx 10$ MeV one should use the temperature T tabulated in ref. [16]. For $E > 10$ MeV, the temperature should be determined from eq. (2.35), again with $y_0 = E$ [27]. In experiment, of course, the effect of the interaction V as a whole is measured. The spreading width (2.1) is obtained by summing eq. (4.5) over k, a and q after dividing through $\rho(E)$. This gives:

$$ \Gamma^{\downarrow}(E) = 2\pi v^2 \sum_{kaq} \binom{d_h}{h_C} \binom{2k-2}{k+a-1} \frac{g(gT)^{2k-1}}{p_+!h_+!p_-!h_-!} \ \exp\left(-\frac{z(z+1)}{2gT}\right) \ . \tag{4.6} $$

Usually a two-body ansatz is made for the interactions between nucleons. We therefore explicitly carry out the summation in eq. (4.6) for this case. The factor $\exp(-z(z+1)/2gT)$ by which the total spectator density deviates from the nuclear level density — close to unity for typical values of T, g and E — is ignored here in order to analyze the general properties of the spreading width. One finds:

$$ \Gamma^{\downarrow}(E) = 2\pi \, v^2 \, g^2 T[2d_h + 5(gT)^2] \ , \tag{4.7} $$

which is a remarkably simple result. We emphasize two aspects of it:

(i) According to ref. [16] this is a constant as a function of E below $E \approx 10$ MeV because the nuclear temperature should then be independent of E. At higher energy the leading term behaves as $E^{3/2}$. Altogether this amounts to quite a weak energy dependence of $\Gamma^{\downarrow}$ for moderate E — especially if compared to the exponential energy dependence of ρ. This result is in qualitative agreement with the systematics of the spreading widths pertaining to isospin violation [2]. Eq. (4.6) shows that the leading energy dependence of $\Gamma^{\downarrow}$ will be $E^{(2K-1)/2}$ if the body rank of V is K instead of two. This demonstrates that the reason for the weak energy dependence of the experimental $\Gamma^{\downarrow}$ is the two-body character of the symmetry breaking interaction: With increasing excitation energy the complexity (in terms of excitons) of the states increases. This decreases the fraction of states that can be connected by an interaction of low rank, hence the local average square $\ll \mathsf{V}^2 \gg$ decreases. Consequently, the product of $\ll \mathsf{V}^2 \gg$ and ρ varies slowly.

(ii) The contributions ${}^k\Gamma^\downarrow$ of the exciton potential (k=1) and the exciton scattering (k=2) to the spreading width are given by the first and second term on the r.h.s. of eq. (4.7), i. e. by the terms proprotional to T and T^3 respectively. Inserting the typical value of $T = 0.5$ MeV for $E \lesssim 10$ MeV [16] and $g \approx A\ (13\ \mathrm{MeV})^{-1}$ as well as $d_h = A$ (which optimizes the DGA), one finds

$$\frac{{}^1\Gamma^\downarrow}{{}^2\Gamma^\downarrow} \approx \begin{cases} 1 & : \quad E \leq 10\,\mathrm{MeV} \\ \\ 10\,\mathrm{MeV}/E & : \quad E > 10\,\mathrm{MeV}. \end{cases} \tag{4.8}$$

Thus the rank one and rank two exciton interactions contribute about equally strongly to $\Gamma^\downarrow$ in the energy range of typical CN reactions.

C. Discussion

Within the framework of the statistical model the spreading width due to an arbitrary K-body force has been calculated for a compound nuclear reaction of an A-body nucleus. The spreading width is the adequate measure for symmetry nonconservation in complex many-body systems [2,28]. It measures the extent to which the states are smeared out due to the presence of the symmetry breaking force. It also appears as the damping width of a simple configuration — such as an isobaric analog state or a giant resonance — into the complex compound nuclear configurations. Definite numbers for a particular interaction can be given if eq. (4.6) is complemented by the calculation of the spectral average of the interaction in K-body space. Obvious applications are the electromagnetic and weak forces, responsible for the breaking of isospin and parity, respectively. Numerical studies in that direction are currently under progress. For the case of isospin breaking a weak dependence of the spreading width on energy and mass number is known [2]. In the present study the energy or temperature dependence has been related to the body rank of the underlying interaction. Since v^2 depends on A, eqs. (4.6) and (4.7) do not exhibit the complete A-dependence of the spreading width, which is expected to be weak. The variation of $\Gamma^\downarrow$ with

temperature can be compared with the results of other authors. Assuming that the spread of a one-exciton configuration (a $1p\ 0h$ state) is proportional to the square of the excitation energy [29,30], Lauritzen *et al.* [31] have concluded a T^3-dependence for the spreading width in general. Since the decay of the one-exciton configuration is caused by ${}_2\mathsf{V}_{(1,3)}$, we indeed find from eq. (4.4) its damping to be proportional to E^2 and eq. (4.7) indeed indicates a T^3-dependence of the spreading width in A-body space (for transitions caused by the strong interaction only). Studying the coupling of surface modes to single particle motion, Esbensen and Bertsch [32] found the "elementary damping" to be proportional to E, which in the framework of Lauritzen *et. al* corresponds to a T^2-dependence of the spreading width. There is nothing analogous in the present results since we did not consider collective motion in the present paper. De Blasio *et. al* [33] report the damping of giant resonances to be independent of the nuclear temperature. Again the reason is that the statistical damping of the present study is different from the damping mechanism considered there. This is also true for the study of spreading properties of isobaric analogue and Gamov-Teller resonances by Colò *et al.* [34]. This paper indicates, however, that the statistical damping must be taken into account in order to explain the widths of the resonances.

Frequently the treatment of parity violation in CN reactions is restricted to the $k = 1$ part of the weak interaction [35–39]. Eq. (4.8) indicates, however, that the contribution of the operators with exciton rank two should be included in the calculation of a local mean square matrix element.

Finally we point out the limitations of our method. The Hartree Fock method and the particle hole formalism rely on a basis of product states. Such an independent particle model can not describe collective phenomena in nuclei. The present results therefore must be modified if applied to reactions that involve collective excitations. A second problem are the effects of the symmetries *respected* by the interaction under study. These effects will of course manifest themselves automatically when the spectral average in the $K-$body space is calculated. The use of partial level densities for the actors that contain all states at a given energy irrespective of further quantum numbers, however, neglects possible *local*

effects of the respected symmetries. We give an example to illustrate this complication. Consider the weak potential ($k = 1$) in a nucleus. This operator connects many-body states that differ by only one single-body configuration. Parity violation and conservation of total angular momentum demand that the single-body states differ in l but not in j. Since single-body states with $\Delta l = \pm 1$ and $\Delta j = 0$ only exist in different shells, they are separated by a relatively large energy interval. Consequently, the present local average suppresses these contributions to the spreading width. The two-exciton part of the weak interaction on the other hand is not subject to this "local selection rule" because it simultaneously changes two single-ecxiton configurations. This fact was first pointed out by Lewenkopf and Weidenmüller [40]. They estimated that the $k = 2$ part of the weak interaction dominates the local mean square matrix element. In section IV B it was found that potential and scattering contribute about equally to the spreading width without taking the local effects of the symmetries into account. Sufficiently elaborate expressions for the single-exciton level densities of the actors before and after the interaction would only overlap in a small energy range, and consequently the convolution of $\rho_{h_+,p_+}(E-y)\rho_{h_-,p_-}(E-y)$ with $\rho(y-E_z)$ would result in a smaller contribution of the potential to the strength.

ACKNOWLEDGEMENTS

M.G. wishes to thank Simon Kalvoda for stimulating discussions and the *Studienstiftung des deutschen Volkes* for support.

APPENDIX A: GOE CORRELATIONS IN THE EXCITON PICTURE

In this appendix, the differences between the exact (3.6) and approximate (3.7) ensemble average are discussed. We find that eq. (3.6) leads to two types of additional correlation that do not appear in its approximation. Considering two examples, it will also be found, however, that these additional terms are of the same order of magnitude as those neglected by invoking the dilute gas approximation. Since the central formulae of section III A and III B

rely on the DGA, it is not necessary and indeed would be inconsistent to take these quantities into account in the evaluation of the strength function.

The correlation coefficient for two operators is

$$\mathcal{C}_{kk'} \equiv \langle {}^{T}_{L}| \ {}_{k}\mathsf{V}_{(a,q)}| {}^{P}_{H}\rangle\langle {}^{T}_{L}| \ {}_{k'}\mathsf{V}_{(a',q')}| {}^{P}_{H}\rangle = \sum_{\substack{\{H_C;H_-P_-;H_+P_+\}\\ \{L_C;L_-T_-;L_+T_+\}}} \tag{A1}$$

$$\langle H_C H_- P_+|\tilde{\mathsf{v}}|H_C H_+ P_-\rangle\langle L_C L_- T_+|\tilde{\mathsf{v}}|L_C L_+ T_-\rangle\langle {}^{T}_{L}| \left(\mathsf{A}^{P_+}_{H_+}\right)^{\dagger} \left(\mathsf{A}^{P_-}_{H_-}\right) | {}^{P}_{H}\rangle\langle {}^{T}_{L}| \left(\mathsf{A}^{T_+}_{L_+}\right)^{\dagger} \left(\mathsf{A}^{T_-}_{L_-}\right) | {}^{P}_{H}\rangle \, .$$

Estimating this expression by its ensemble average, we obtain:

$$\mathcal{C}_{kk'} = v^2 \sum_{\substack{\{H_C;H_-P_-;H_+P_+\}\\ \{L_C;L_-T_-;L_+T_+\}}} \delta^{L_C L_- T_+}_{H_C H_- P_+} \delta^{L_C L_+ T_-}_{H_C H_+ P_-} \langle {}^{T}_{L}| \left(\mathsf{A}^{P_+}_{H_+}\right)^{\dagger} \left(\mathsf{A}^{P_-}_{H_-}\right) | {}^{P}_{H}\rangle\langle {}^{T}_{L}| \left(\mathsf{A}^{T_+}_{L_+}\right)^{\dagger} \left(\mathsf{A}^{T_-}_{L_-}\right) | {}^{P}_{H}\rangle \ . \tag{A2}$$

Since hole and particle configurations by definition do not intersect, the Kronecker symbol implies $p_+ = t_+$ and $p_- = t_-$. Addition and subtraction of these equations yields $q = q'$ and $a = a'$. In other words: two operators that differ only by their rank are correlated. This is the first difference to the ensemble average (3.7) used in section III A, were it was found that different operators are uncorrelated. The second type of additional correlation we find in eq. (A2) results from the intricate restriction pattern in the sum over hole configurations: The restriction only applies to the indices *within* the groups L_C and L_-. The condition for nonzero correlations, however, is that the set $L_C L_-$ of $l_C + l_-$ indices coincide with the set $H_C H_-$ of $h_C + h_-$ indices. We adress these two cases one after the other. First, let k' be equal to $k + \xi$ with $\xi > 0$. The sum over T_+, T_- may now readily be carried out:

$$\mathcal{C}_{k,k+\xi} = v^2 \sum_{\substack{\{H_C;H_-P_-;H_+P_+\}\\ \{L_C;L_-;L_+\}}} \delta^{L_C L_-}_{H_C H_-} \delta^{L_C L_+}_{H_C H_+} \langle {}^{T}_{L}| \left(\mathsf{A}^{P_+}_{H_+}\right)^{\dagger} \left(\mathsf{A}^{P_-}_{H_-}\right) | {}^{P}_{H}\rangle\langle {}^{T}_{L}| \left(\mathsf{A}^{P_+}_{L_+}\right)^{\dagger} \left(\mathsf{A}^{P_-}_{L_-}\right) | {}^{P}_{H}\rangle \ . \tag{A3}$$

This expression may be simplified rather easily if we consider the special case of $l_C = 0$. Then $H_C = \Xi$ and we obtain

$$\mathcal{C}_{k,k+\xi} = v^2 \sum_{\{\Xi;H_-P_-;H_+P_+\}} \langle {}^{T}_{L}| \left(\mathsf{A}^{P_+}_{H_+}\right)^{\dagger} \left(\mathsf{A}^{P_-}_{H_-}\right) | {}^{P}_{H}\rangle\langle {}^{T}_{L}| \left(\mathsf{A}^{P_+}_{\Xi H_+}\right)^{\dagger} \left(\mathsf{A}^{P_-}_{\Xi H_-}\right) | {}^{P}_{H}\rangle \ . \tag{A4}$$

For further simplification of this expression let us introduce the product of hole number operators

$$(\mathsf{L}_\Xi) \equiv \mathsf{a}^\dagger_{\beta_1}\mathsf{a}_{\beta_1}\ldots\mathsf{a}^\dagger_{\beta_\xi}\mathsf{a}_{\beta_\xi} \tag{A5}$$

for $\Xi = (\beta_1 \ldots \beta_\xi)$. This operator has the property

$$\sum_{\{\Xi\}}(\mathsf{L}_\Xi)|\,{}^P_H\rangle = \binom{h}{\xi}|\,{}^P_H\rangle \ , \tag{A6}$$

where a term containing the factorial of a negative number is understood to be zero. Note that

$$|\langle\,{}^T_L|\left(\mathsf{A}^{P_+}_{\Xi H_+}\right)^\dagger\left(\mathsf{A}^{P_-}_{\Xi H_-}\right)|\,{}^P_H\rangle| = |\langle\,{}^T_L|\left(\mathsf{A}^{P_+}_{H_+}\right)^\dagger\mathsf{L}_\Xi\left(\mathsf{A}^{P_-}_{H_-}\right)|\,{}^P_H\rangle| \ . \tag{A7}$$

One therefore obtains

$$|\mathcal{C}_{k,k+\xi}| = v^2\binom{h-h_-}{\xi}\sum_{\{H_-P_-;\,H_+P_+\}}\langle\,{}^T_L|\left(\mathsf{A}^{P_+}_{H_+}\right)^\dagger\left(\mathsf{A}^{P_-}_{H_-}\right)|\,{}^P_H\rangle^2 \ . \tag{A8}$$

The quantity $h - h_-$ is the number of hole spectators, cf. section III A. For the correlation coefficient of an operator with itself, we find

$$\mathcal{C}_{kk} = v^2\binom{d_h}{\xi}\sum_{\{H_-P_-;\,H_+P_+\}}\langle\,{}^T_L|\left(\mathsf{A}^{P_+}_{H_+}\right)^\dagger\left(\mathsf{A}^{P_-}_{H_-}\right)|\,{}^P_H\rangle^2 \ , \tag{A9}$$

so that

$$\frac{|\mathcal{C}_{k,k+\xi}|}{\mathcal{C}_{kk}} = \frac{(h-h_-)!(d_h-\xi)!}{(h-h_--\xi)!d_h!} \ . \tag{A10}$$

Under the assumption that $K \ll A = d_h$, which implies $\xi \ll d_h$, this yields

$$\frac{|\mathcal{C}_{k,k+\xi}|}{\mathcal{C}_{kk}} \lesssim \left(\frac{h-h_-}{d_h}\right)^\xi \ . \tag{A11}$$

This type of correlation may therefore be neglected if the number of spectators (in hole space) is small compared to the dimension d_h of the single hole space. This is certainly true if $h \ll d_h$, which is the condition for the DGA introduced in section III A.

The second type of additional correlation appearing in eq. (A2) shall be illustrated using the operator ${}_1\mathsf{V}_{(0,0)}$ of a two body interaction (cf. Tab. I). We find for its correlation coefficient:

$$\mathcal{C} = v^2 \sum_{\substack{\beta\gamma\delta \\ \beta'\gamma'\delta'}} \delta^{\beta\gamma}_{\beta'\gamma'}\delta^{\beta\delta}_{\beta'\delta'} \langle {}^{T}_{L}|\mathsf{a}^\dagger_\delta \mathsf{a}_\gamma|{}^{P}_{H}\rangle \langle {}^{T}_{L}|\mathsf{a}^\dagger_{\delta'} \mathsf{a}_{\gamma'}|{}^{P}_{H}\rangle$$
$$= v^2 \sum_{\gamma\delta} \langle {}^{T}_{L}|\mathsf{a}^\dagger_\delta \mathsf{a}_\gamma|{}^{P}_{H}\rangle \left(\langle {}^{T}_{L}|\mathsf{a}^\dagger_\delta \mathsf{a}_\gamma|{}^{P}_{H}\rangle d_h + \langle {}^{T}_{L}|{}^{P}_{H}\rangle h - 2\langle {}^{T}_{L}|\mathsf{a}^\dagger_\delta \mathsf{a}_\gamma|{}^{P}_{H}\rangle \right) . \tag{A12}$$

In the brackets, the first term is of the type discussed in sections III A and III B. The second term is suppressed relative to the first one by a factor h/d_h and by the fact that it contributes only to diagonal elements. The third term is suppressed by a factor $2/d_h$. Obviously, it is justified to neglect the second and third term as long as the DGA makes sense. Since out of the nine operators of a two-body interaction which actually contribute to the strength function ${}_1\mathsf{V}_{(0,0)}$ is the only one that shows correlations of this type, eq. (3.7) seems to be a very good approximation.

APPENDIX B: PROOF OF EQUATION (3.9)

In eq. (3.8), consider the special case of $p_+ = h_+ = h_- = 0$:

$$\mathcal{M} \equiv \sum_{\{P\}} \langle {}^{T}_{H}| \left(\mathsf{A}_0^{P_-} \right) |{}^{P}_{H}\rangle^2 f(P) . \tag{B1}$$

Let P, P_- be

$$P = (r_1 \dots r_p) \qquad \text{and} \qquad P_- = (s_1 \dots s_{p_-}) \tag{B2}$$

and f a function that is completely symmetric in the arguments $r_1 \dots r_p$. The restricted sum of eq. (B1) can be written as the unrestricted sum

$$\mathcal{M} = \frac{1}{p!} \sum_{P} \langle {}^{T}_{H}|\mathsf{a}_{s_1} \dots \mathsf{a}_{s_{p_-}} \mathsf{a}^\dagger_{r_p} \dots \mathsf{a}^\dagger_{r_1}|{}^{0}_{H}\rangle^2 \quad f(P) . \tag{B3}$$

The matrix elements vanish unless the $s_1 \dots s_{p_-}$ all appear in $r_1 \dots r_p$. Consider a term that satisfies this condition. There are $\binom{p}{p_-}$ ways in which the indices r that agree with one of $s_1 \dots s_{p_-}$ can be distributed over the postions $1 \dots p$. Therefore, imposing the requirement that the first p_- indices $r_1 \dots r_{p_-}$ should agree with $s_1 \dots s_{p_-}$ (up to a permutation) one obtains

$$\mathcal{M} = \frac{1}{p!}\binom{p}{p_-} \sum_{\substack{t_1\dots t_{p-p_-} \\ \neq s_1\dots s_{p_-}}} \sum_{r_1\dots r_{p_-}} \langle {}^{T}_{H}|\mathsf{a}_{s_1}\dots\mathsf{a}_{s_{p_-}}\mathsf{a}^\dagger_{t_{p-p_-}}\dots\mathsf{a}^\dagger_{t_1}\mathsf{a}^\dagger_{r_{p_-}}\dots\mathsf{a}^\dagger_{r_1}|\,{}^{0}_{H}\rangle^2 \quad f(t_1\dots t_{p-p_-}, P_-)$$

$$= \frac{1}{p!}\binom{p}{p_-}(p-p_-)!p_-! \sum_{\substack{\{P-p_-\} \\ \neq P_-}} \langle {}^{T}_{H}|\mathsf{a}^\dagger_{t_{p-p_-}}\dots\mathsf{a}^\dagger_{t_1}|\,{}^{0}_{H}\rangle^2 \quad f(P-p_-, P_-)\ . \tag{B4}$$

Here, the restriction $t_1\dots t_{p-p_-} \neq s_1\dots s_{p_-}$ means that none of the indices t_i ,$i = 1\dots p-p_-$, is allowed to coincide with any one of the indices s_k, $k = 1\dots p_-$. The short hand notation $\{P-p_-\} \neq P_-$ means the same. Eq. (B4) is obviously the same as

$$\sum_{\substack{\{P-p_-\} \\ \neq P_-}} \langle {}^{T}_{H}|\,{}^{P-p_-}_{H}\rangle^2 \quad f(P-p_-, P_-) = \sum_{\substack{\{P-p_-\} \\ \neq P_-}} \delta^T_{P-p_-} \quad f(P-p_-, P_-)\ , \tag{B5}$$

which is easily generalized to eq. (3.9).

REFERENCES

[1] C. M. Frankle, S. J. Seestrom, Yu. P. Popov, E. I. Sharapov, and N. R. Robertson *Phys. Part. Nucl.* **24**, 401 (1993)

[2] J. Reiter and H.L. Harney, *Z. Phys. A* **337**, 121 (1990)

[3] M. Granzow, H. L. Harney, and H. Kalka, *Phys. Lett.* **B 331**, 257 (1994)

[4] S. S. M. Wong, *Statistical Nuclear Spectroscopy*, Oxford U.P., Oxford, 1986, Chapter II

[5] Peter Ring and Peter Schuck: *The Nuclear Many Body Problem*, Springer Verlag, New York, 1980

[6] Alexander L. Fetter and John Dirk Walecka, *Quantum Theory of Many Particle Systems*, Chapter 56, McGraw–Hill, New York, 1971

[7] Aage Bohr and Ben Mottelson: *Nuclear Structure* Vol I, W. A. Benjamin, New York, 1969, p. 373ff. Note the misprint in the sign of eq. (3b-27i).

[8] Usually the holes are defined as the time reversed state of the corresponding body states and phases are introduced in order to ensure the desired transformation properties. We omit these complications since they do not affect our arguments.

[9] Frank Harris, Hendrik Monkhorst, David Freeman: *Algebraic and Diagrammatic Methods in Many Fermion Theory*, Oxford U.P., Oxford, 1992, Chapter 5

[10] We do not give the sign of the operator, since it is irrelevant for our purposes.

[11] T. O. E. Ericson, *Adv. Phys.* **9**, 425 (1960)

[12] A. Gilbert and A. G. W. Cameron, *Can. J. Phys.* **43**, 1446 (1965)

[13] R. G. Stokstad, in *Treatise on Heavy Ion Science, Vol 3*, p.83ff, Editor: D. Broomeley, Plenum, New York, 1985

[14] Ettore Gadioli and Peter Edward Hodgson, *Preequilibrium Nuclear Reactions*, Ch. 2,

Oxford UP, 1992

[15] Victor F. Weisskopf and J. M. Blatt, *Theoretical Nuclear Physics*, p.367f, Wiley, New York, 1960

[16] T. v. Egidy, H. H. Schmidt, and A. N. Behkami *Nucl. Phys.* **A481**, 189 (1988)

[17] F. C. Williams *Nucl. Phys.* **A166**, 231 (1971)

[18] P. Obložinský, *Nucl. Phys.* **A453**, 127 (1986)

[19] D. Lovelock and H. Rund, *Tensors, differential forms, and variational principles*, Section 4.2, Dover, New York, 1989

[20] J. J. Griffin, *Phys. Rev. Lett.* **17**, 478 (1966) and *Phys. Lett.* **B24**, 5, (1967)

[21] C. K. Cline, *Nucl. Phys.* **A193**, 417 (1972)

[22] I. Ribanský, P. Obložinský, and E. Běták, *Nucl. Phys.* **A205**, 545 (1973)

[23] M. Blann, *Annu. Rev. Nucl. Sci.* **25**, 123 (1975)

[24] H. Nishioka, J. J. M. Verbaarschot, H. A. Weidenmüller, and S. Yoshida *Ann. Phys.* (NY) **172**, 67 (1986)

[25] F. C. Williams, *Phys. Lett.* **B31**, 184 (1970)

[26] P. Obložinský, I. Ribanský, and E. Běták, *Nucl. Phys.* **A226**, 347 (1974)

[27] In order to better respect the variation of T with energy, one could choose y_0 as the maximum of the integrand in eq. (3.21). The maximum is, however, close to the excitation energy E.

[28] D. Boosé, H. L. Harney, and H. A. Weidenmüller, *Phys. Rev. Lett.* **56**, 2012 (1986)

[29] G. F. Bertsch, P. A. Broglia, and P. F. Bortignon *Rev. Mod. Phys.* **55**, 287,(1983), p. 290

[30] C. Mahaux and R. Sartor: *Single-particle motion in nuclei* in *Advances in Nuclear Physics*, Vol. 20, p. 162f; Editors: J. W. Negele and Erich W. Vogt, Plenum, New York 1991

[31] B. Lauritzen, T. Døssing, and R. A. Broglia *Nucl. Phys.* **A457**, 61 (1986), Appendix A

[32] H. Esbensen and G. F. Bertsch *Phys. Rev. Lett.* **52**, 2257 (1984)

[33] F. V. De Blasio, W. Cassing, M. Tohyama, P. F. Bortignon, and R. A. Broglia *Phys. Rev. Lett.* **68**, 1663 (1992)

[34] G. Coló, N. Van Giai, P. F. Bortignon, and R. A. Broglia, *Phys. Rev. C* **50**, 1496 (1994)

[35] E. G. Adelberger, and W. C. Haxton, *Annu. Rev. Nucl. Part. Sci.* **35**, 50 (1985)

[36] S. E. Koonin, C. W. Johnson, and P. Vogel, *Phys. Rev. Lett.* **69**, 1163 (1992)

[37] N. Auerbach, *Phys. Rev. C* **45**, R541 (1992)

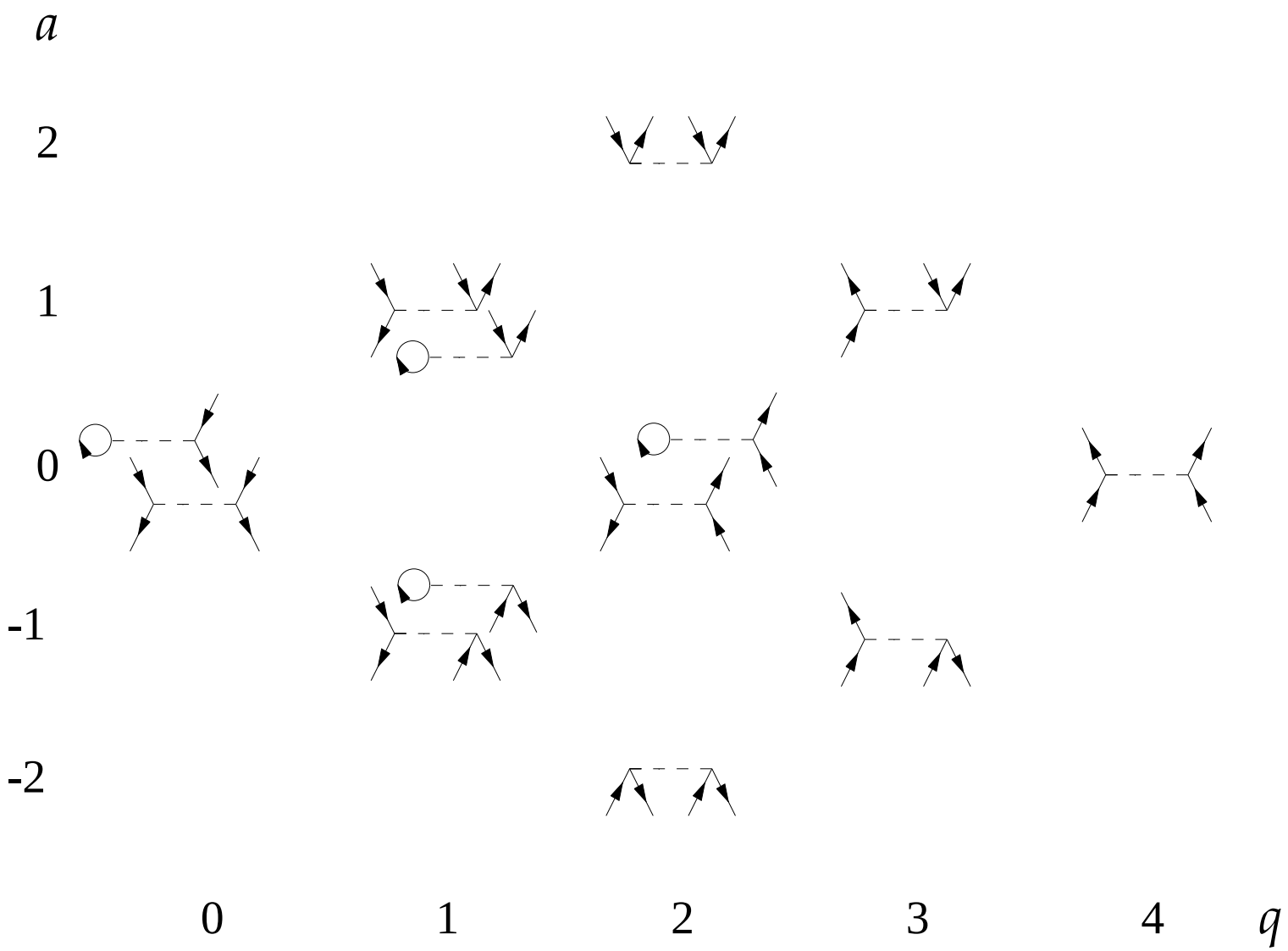
[38] J. D. Bowman, G. T. Garvey, Mikkel B. Johnson, and G. E. Mitchell *Annu. Rev. Nucl. Part. Sci.* **43**, 829 (1993)

[39] V. V. Flambaum and O. K. Vorov, *Phys. Rev. Lett.* **70**, 4051 (1993)

[40] C. H. Lewenkopf and H. A. Weidenmüller *Phys. Rev. C* **46**, 2601 (1992)

FIGURES

FIG. 1. Two-body interaction in the exciton representation: Diagrams with $k = 1, 2$.


a
2
1
0
-1
-2
0
1
2
3
4
q

TABLES

TABLE I. Two-body interaction in the exciton picture.

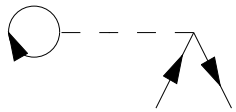
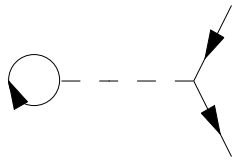
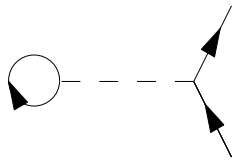
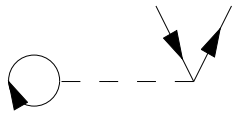
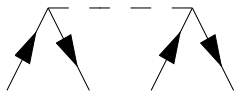
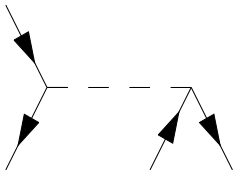
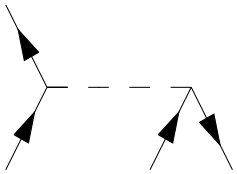
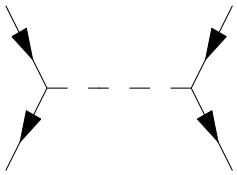
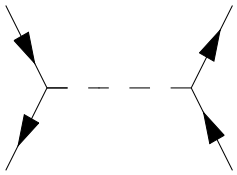
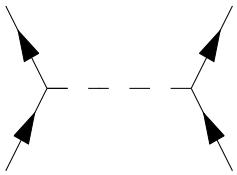
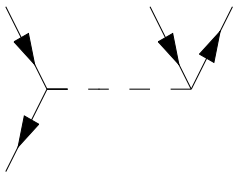
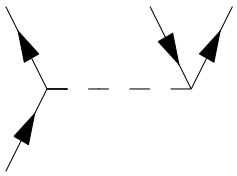
Classification	Operator	Diagram
${}_0V_{(0,0)}$	$+\sum_{\{\beta\gamma\}}\langle\beta\gamma\|\tilde{v}\|\beta\gamma\rangle$	
${}_1V_{(-1,1)}$	$+\sum_{\beta\gamma s}\langle\beta\gamma\|\tilde{v}\|\beta s\rangle a_\gamma a_s$	
${}_1V_{(0,0)}$	$-\sum_{\beta\delta\gamma}\langle\beta\delta\|\tilde{v}\|\beta\gamma\rangle a^\dagger_\gamma a_\delta$	
${}_1V_{(0,2)}$	$+\sum_{\beta ts}\langle\beta s\|\tilde{v}\|\beta t\rangle a^\dagger_s a_t$	
${}_1V_{(1,1)}$	$+\sum_{\beta\gamma s}\langle\beta s\|\tilde{v}\|\beta\gamma\rangle a^\dagger_s a^\dagger_\gamma$	
${}_2V_{(-2,2)}$	$+\sum_{\{\beta\gamma\}\{st\}}\langle\beta\gamma\|\tilde{v}\|st\rangle a_\gamma a_\beta a_s a_t$	
${}_2V_{(-1,1)}$	$+\sum_{\{\beta\delta\}s\gamma}\langle\beta\delta\|\tilde{v}\|\gamma s\rangle a^\dagger_\gamma a_\delta a_\beta a_s$	

TABLE I. *(Continued)* Two-body interaction in the exciton picture.

Classification	Operator	Diagram
${}_2\mathsf{V}_{(-1,1)}$	$+\sum_{\beta\{su\}t}\langle\beta t\|\tilde{\mathrm{v}}\|su\rangle \mathsf{a}_t^\dagger \mathsf{a}_\beta \mathsf{a}_s \mathsf{a}_u$	
${}_2\mathsf{V}_{(0,0)}$	$+\sum_{\{\gamma\epsilon\}\{\beta\delta\}}\langle\gamma\epsilon\|\tilde{\mathrm{v}}\|\beta\delta\rangle \mathsf{a}_\beta^\dagger \mathsf{a}_\delta^\dagger \mathsf{a}_\epsilon \mathsf{a}_\gamma$	
${}_2\mathsf{V}_{(0,2)}$	$-\sum_{\gamma t\beta s}\langle\gamma s\|\tilde{\mathrm{v}}\|\beta t\rangle \mathsf{a}_s^\dagger \mathsf{a}_\beta^\dagger \mathsf{a}_\gamma \mathsf{a}_t$	
${}_2\mathsf{V}_{(0,4)}$	$+\sum_{\{tv\}\{su\}}\langle su\|\tilde{\mathrm{v}}\|tv\rangle \mathsf{a}_u^\dagger \mathsf{a}_s^\dagger \mathsf{a}_t \mathsf{a}_v$	
${}_2\mathsf{V}_{(1,1)}$	$+\sum_{\delta\{\beta\gamma\}s}\langle\delta s\|\tilde{\mathrm{v}}\|\beta\gamma\rangle \mathsf{a}_s^\dagger \mathsf{a}_\beta^\dagger \mathsf{a}_\gamma^\dagger \mathsf{a}_\delta$	
${}_2\mathsf{V}_{(1,3)}$	$+\sum_{u\beta\{st\}}\langle st\|\tilde{\mathrm{v}}\|\beta u\rangle \mathsf{a}_t^\dagger \mathsf{a}_s^\dagger \mathsf{a}_\beta^\dagger \mathsf{a}_u$	
${}_2\mathsf{V}_{(2,2)}$	$+\sum_{\{\beta\gamma\}\{st\}}\langle st\|\tilde{\mathrm{v}}\|\beta\gamma\rangle \mathsf{a}_t^\dagger \mathsf{a}_s^\dagger \mathsf{a}_\beta^\dagger \mathsf{a}_\gamma^\dagger$